From (4) and (6′), all the odd ordered derivatives of $f(x)$ are zero when $x = 0$. By successive applications of (6′) we have (since $f''(0) = \pm 2c^2$)

$$f^{\text{IV}}(0) = 2c^4, \quad f^{\text{VI}}(0) = \pm 2c^6, \cdots, f^{(4n)}(0) = 2c^{4n}, \quad f^{(4n+2)}(0) = \pm 2c^{4n+2}.$$

Hence, by Maclaurin's theorem,

$$f(x) = 2 \pm 2c^2x^2/2! + 2c^4x^4/4! \pm 2c^6x^6/6! + \cdots.$$

This gives either

$$f(x) = 2 \cdot \cosh cx, \quad \text{or} \quad f(x) = 2 \cdot \cos cx,$$

to which solutions $f(x) = 0$ should be added as before.

Also solved by H. C. FEEMSTER, OSCAR S. ADAMS, PAUL CAPRON, and the PROPOSER.

MECHANICS.

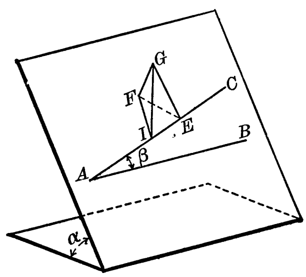
319. Proposed by LAENAS G. WELD, Pullman, Illinois.

A hexagonal pencil lies upon the inclined top of a drawing table and is on the point of either rolling or sliding. Find the angle between its direction and the horizontal edge of the table, the coefficient of friction being μ.

SOLUTION BY H. S. UHLER, Yale University.

The condition for being on the verge of sliding is expressed by $\mu = \tan \alpha$, where α denotes the angle between the table top and the horizontal and is sometimes called the "limiting angle of repose." The proof of this relation seems superfluous in this place because it is given in practically all text-books which are devoted in part or entirely to elementary mechanics.

In order that the pencil may be on the point of rolling, it is obviously necessary and sufficient that the vertical line through the center of gravity intersect the lateral edge which is at the lowest level. In the diagram let $\overline{AB}$ and $\overline{AC}$ indicate respectively a horizontal line on the table top and the lowest lateral edge of the hexagonal prism. G marks the center of gravity which is assumed to lie in the geometric axis of the pencil. $\overline{GI}$ denotes the vertical through G, that is,



the line of action of the weight of the pencil which intersects the edge $\overline{AC}$ in the point I. $\overline{GE}$ and $\overline{GF}$ are perpendiculars dropped from G upon the edge $\overline{AC}$ and the table top, respectively. In other words, the plane EFG contains a right section of the prism passing through the center of gravity. Without quoting the well-known theorems of elementary geometry we see at once that $\angle EFI = \beta$, $\angle EGF = 30°$, $\angle FGI = \alpha$, $\angle EFG = \angle FEI = \angle GFI = 90°$. Consequently, $\overline{FG}/\overline{EF} = \cot 30° = \sqrt{3}$, $\overline{EF}/\overline{FI} = \cos \beta$, and $\overline{FI}/\overline{FG} = \tan \alpha$. Multiplying these three equations together we obtain $1 = \sqrt{3} \cos \beta \tan \alpha$, but $\tan \alpha = \mu$, hence

$$\beta = \cos^{-1}\left(\frac{1}{\sqrt{3}\mu}\right).$$

Remark.—Since the greatest value of a cosine is unity the least value of μ is equal to $1/\sqrt{3} \doteq 0.57735$ corresponding to $\alpha = 30°$ and $\beta = 0°$, as it should be.

Also solved by W. J. THOME, M. R. BOWERMAN, and GEORGE PAASWELL.